Infinite Symmetries and Ward Identities in Celestial Holography

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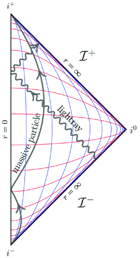
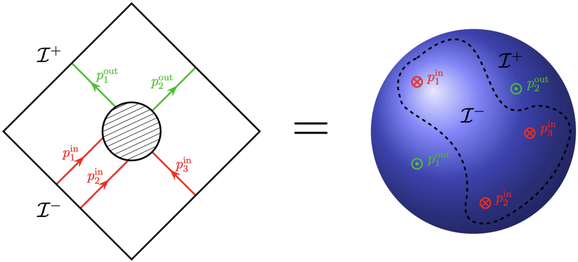
Holographic Principle

- AdS/CFT is wonderful!
  - An exact duality between quantum gravity in AdS and gauge theory
  - Tested precisely and remarkable agreements found on two sides
  - Profound implications on quantum gravity:
    e.g. entanglement, black hole information paradox, ...
  - ...
- Holographic principle motivated by BH entropy ['t Hooft '93; Susskind '94]
  ★ Quantum gravity is holographic in general!
- Flat holography



  - Natural generalization but difficult
    due to null and non-smooth boundary
  - Celestial holography? [Strominger ...]

Celestial holography: what and why?



I will focus on massless fields in 4D.

[figure adapted from Strominger '17]

Celestial holography [Cheung,Fuente,Sundrum '16, Pasterski,Shao,Strominger '16]

4D S-matrix = 2D CFT correlator

- A new paradigm for flat holography beyond AdS/CFT
- A new program of reformulating scattering amplitude
- New insights into classical GR and quantum gravity

Motivation: what are the full underlying symmetries?

Symmetry is one of the most important guiding principles in physics!

The whole story of celestial holography is about symmetry.

It was motivated from and began with symmetries ....

soft theorems $\leftrightarrow$ BMS asymptotic symmetries

momentum space amplitude $\leftrightarrow$ manifest translational symmetry

celestial amplitude $\leftrightarrow$ manifest Lorentz symmetry

...but symmetries are still not fully understood!

Q: What are the full underlying symmetries governing the amplitudes?

Outline

Introduction

Kinematics

- In 4D, the null momentum can be parametrized as

$$p^\mu = \epsilon \omega q^\mu, \qquad q^\mu = \Big(1 + z\bar{z}, \quad z + \bar{z}, \quad -i(z - \bar{z}), \quad 1 - z\bar{z}\Big)$$

where $\epsilon = \pm 1$ for out-going/in-coming particle, and $z, \bar{z} \in S^2$ are coordinates on the *celestial sphere*.

- Under $SL(2, \mathbb{C})$

$$z \to \frac{az + b}{cz + d}, \qquad \bar{z} \to \frac{\bar{a}\bar{z} + \bar{b}}{\bar{c}\bar{z} + \bar{d}}, \quad ad - bc = 1$$

$$\rightsquigarrow \qquad p^\mu \to \Lambda^\mu{}_\nu p^\nu, \qquad \omega \to |cz + d|^2 \omega$$

- Lorentz group $SO(3,1)$ in the bulk

  $\rightsquigarrow$ conformal group $SL(2, \mathbb{C})$ on the celestial sphere

- Polarization

  gluon : $$\varepsilon^\mu_+(q) = \frac{1}{\sqrt{2}} \partial_z q^\mu, \qquad \varepsilon^\mu_-(q) = \frac{1}{\sqrt{2}} \partial_{\bar{z}} q^\mu$$

  graviton : $$\varepsilon^{\mu\nu}_\pm = \varepsilon^\mu_\pm \varepsilon^\nu_\pm$$

Celestial Amplitude

- Scattering amplitudes in momentum space are manifested translational invariant.

$$\mathcal{A}_n(J_i, p_i^\mu) = A_n(J_i, p_i^\mu)\ \delta^4\Big(\sum_i p_i^\mu\Big)\ , \qquad p_i^\mu = \epsilon_i \omega_i q_i^\mu\ ,$$

- Mellin transformation gives **celestial amplitude**

[Pasterski, Shao, Strominger '17]

$$\begin{aligned}\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) &= \Big(\prod_{j=1}^{n} \int_0^\infty d\omega_j\ \omega_j^{\Delta_j - 1}\Big) \mathcal{A}_n(J_i, p_i^\mu) \\ &= \langle \mathcal{O}^{\epsilon_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}^{\epsilon_n}_{\Delta_n, J_n}(z_n, \bar{z}_n) \rangle_{\text{CCFT}}\end{aligned}$$

- Momentum eigen-state $|p\rangle \rightsquigarrow$ boost eigen-state $|\Delta, z, \bar{z}\rangle$
- Further introduce fermionic coordinates $\rightsquigarrow$ celestial superamplitudes

[Jiang '21][Brandhuber, Brown, Gowdy, Spence, Travaglini '21]

- Soft theorem $\rightarrow$ Ward identity
  Collinear factorization $\rightarrow$ Celestial OPE

Global symmetry: Poincare, conformal and superconformal

Poincare symmetry of momentum space amplitude

$$\mathcal{A}_n(J_i, p_i^\mu) = A_n(J_i, p_i^\mu)\ \delta^4(\sum_i p_i^\mu)\ , \qquad p_i^\mu = \varepsilon_i \omega_i q_i^\mu\ ,$$

- The amplitude should be invariant under translation and Lorentz transformation:

$$\sum_j \varepsilon_j P_j^\mu \mathcal{A}_n(J_i, p_i^\mu) = 0\ , \qquad \sum_j M_j^{\mu\nu} \mathcal{A}_n(J_i, p_i^\mu) = 0\ ,$$

where

$$P^\mu = \varepsilon p^\mu = \omega q^\mu\ , \qquad M^{\mu\nu} = p^\mu \frac{\partial}{\partial p_\nu} - p^\nu \frac{\partial}{\partial p_\mu}\ .$$

- Translational symmetry manifest, but Lorentz symmetry obscure

Poincare symmetry of celestial amplitude

$$\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) = \Big(\prod_{j=1}^{n}\int_0^\infty d\omega_j\ \omega_j^{\Delta_j - 1}\Big)\mathcal{A}_n(J_i, p_i^\mu)\ , \quad p^\mu = \varepsilon\omega q^\mu(z,\bar{z})\ .$$

- Poincare invariance of celestial ampllitude:

$$\sum_j \{\ \varepsilon_j P_j^\mu\ , \quad L_j^k\ , \quad \bar{L}_j^k\ \}\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) = 0\ ,$$

where

$$L^k = h(k+1)z^k + z^{k+1}\partial_z\ , \quad \bar{L}^k = \bar{h}(k+1)\bar{z}^k + z^{k+1}\partial_{\bar{z}}\ ,$$

$$P^\mu = q^\mu e^{\partial_\Delta} = q^\mu e^{(\partial_h + \partial_{\bar{h}})/2} \rightsquigarrow \text{obscure}. \qquad k = 0, \pm 1$$

- $L^k, \bar{L}^k$ satisfy the algebra of $SL(2,\mathbb{C}) \simeq SO(3,1)$

$$[L^k, L^l] = -(k-l)L^{k+l}\ , \qquad [\bar{L}^k, \bar{L}^l] = -(k-l)\bar{L}^{k+l}$$

Celestial amplitude as CFT correlator

- Celestial amplitudes transform as conformal correlators under $SL(2,\mathbb{C})$

$$\mathcal{M}_n\Big(\Delta_i, J_i, \frac{az_i+b}{cz_i+d}, \frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}}\Big) = \Big(\prod_{j=1}^{n}(cz_j+d)^{\Delta_j+J_j}(\bar{c}\bar{z}_j+\bar{d})^{\Delta_j-J_j}\Big)\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i)$$

- Lorentz symmetry $\rightsquigarrow$ Celestial amplitude = CFT correlator

$$\mathcal{M}_n(\Delta_i, J_i, z_i, \bar{z}_i) = \langle \mathcal{O}^{\varepsilon_1}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}^{\varepsilon_n}_{\Delta_n,J_n}(z_n,\bar{z}_n)\rangle_{\text{CCFT}}$$

$$\Delta = h+\bar{h}\ , \qquad J = h-\bar{h}$$ = spin in 2D = helicity in 4D

- To form a complete basis, the dimension should reside in the principal series of $SL(2,\mathbb{C})$

$$\Delta = 1+i\lambda\ , \qquad \lambda\in\mathbb{R}\ ,$$

but it is useful to analytically continue to the whole complex plane.

Bulk conformal symmetry of celestial amplitude

- For null momentum of massless particle $\rightsquigarrow$ spinor-helicity formalism

$$p^{\alpha\dot\alpha} = \sigma_\mu^{\alpha\dot\alpha} p^\mu = \lambda^\alpha \tilde\lambda^{\dot\alpha} \ , \qquad \alpha, \dot\alpha = 1, 2$$

- By introducing

$$\mathsf{p}^\alpha = \begin{pmatrix} 1 \\ z \end{pmatrix} e^{\partial_h/2} \ , \qquad \tilde{\mathsf{p}}^{\dot\alpha} = \begin{pmatrix} 1 \\ \bar z \end{pmatrix} e^{\partial_{\bar h}/2} \ ,$$

$$\mathsf{k}^\alpha = \begin{pmatrix} \partial_z \\ z\partial_z + 2h - 1 \end{pmatrix} e^{-\partial_h/2} \ , \qquad \tilde{\mathsf{k}}^{\dot\alpha} = \begin{pmatrix} \partial_{\bar z} \\ \bar z\partial_{\bar z} + 2\bar h - 1 \end{pmatrix} e^{-\partial_{\bar h}/2} \ ,$$

we can construct the conformal generators

$$P^{\alpha\dot\alpha} = \frac{1}{2}\sigma_\mu^{\alpha\dot\alpha} P^\mu = \mathsf{p}^\alpha \tilde{\mathsf{p}}^{\dot\alpha} \ , \qquad K^{\alpha\dot\alpha} = \frac{1}{2}\sigma_\mu^{\alpha\dot\alpha} K^\mu = \mathsf{k}^\alpha \tilde{\mathsf{k}}^{\dot\alpha} \ .$$

$$L^{\alpha\beta} = \frac{1}{2}(\mathsf{k}^\alpha \mathsf{p}^\beta + \mathsf{k}^\beta \mathsf{p}^\alpha) \ , \qquad L^{\dot\alpha\dot\beta} = \frac{1}{2}(\tilde{\mathsf{k}}^{\dot\alpha} \tilde{\mathsf{p}}^{\dot\beta} + \tilde{\mathsf{k}}^{\dot\beta} \tilde{\mathsf{p}}^{\dot\alpha}) \ ,$$

$$D = 1 - (h + \bar h) = 1 - \Delta \ .$$

- Their commutators satisfy the conformal algebra $\mathfrak{so}(4,2)$.

Superconformal symmetry & $\mathcal{N}=4$ SYM

- $\mathcal{N}=4$ SYM theory enjoys the maximally superconformal symmetry $\mathfrak{psu}(2,2|4)$
$\rightsquigarrow$ tremendous progress on the scattering amplitude in $\mathcal{N}=4$ SYM theory: very high loop, underlying mathematical structure....!

- All the on-shell degrees of freedom can be packaged into an on-shell superfield by introducing Grassmann variables η^A, $A=1,2,3,4$:

$$\begin{aligned}\Psi(p,\eta) \quad = \quad & \mathsf{G}^+(p)+\eta^A\Gamma_A(p)+\frac{1}{2!}\eta^A\eta^B\Phi_{AB}(p) \\ & +\frac{1}{3!}\epsilon_{ABCD}\eta^A\eta^B\eta^C\bar{\Gamma}^D(p)+\frac{1}{4!}\epsilon_{ABCD}\eta^A\eta^B\eta^C\eta^D\mathsf{G}^-(p)\ .\end{aligned}$$

particle	G^+	G^-	$\Phi_{AB}=-\Phi_{BA}$	Γ_A	$\bar{\Gamma}^A$
helicity J	$+1$	-1	0	$+\frac{1}{2}$	$-\frac{1}{2}$
on-shell d.o.f.	1	1	6	4	4
$SU(4)_R$	singlet $(\mathbf{1})$	singlet $(\mathbf{1})$	anti-symmetric $(\mathbf{6})$	fundamental $(\mathbf{4})$	anti-fundamental $(\bar{\mathbf{4}})$

- Superamplitude

$$\mathscr{A}_n(p_i,\eta_i)=\langle\Psi(p_1,\eta_1)\cdots\Psi(p_n,\eta_n)\rangle\ .$$

Superconformal symmetry in $\mathcal{N}=4$ SYM

- The superconformal generators are given by

$$\mathsf{P}^{\alpha\dot\alpha}=\lambda^\alpha\tilde\lambda^{\dot\alpha}\,,\qquad \mathsf{K}^{\alpha\dot\alpha}=\frac{\partial}{\partial\lambda_\alpha}\frac{\partial}{\partial\tilde\lambda_{\dot\alpha}}\,,\qquad \mathsf{D}=\frac{1}{2}\lambda^\alpha\frac{\partial}{\partial\lambda^\alpha}+\frac{1}{2}\tilde\lambda^{\dot\alpha}\frac{\partial}{\partial\tilde\lambda^{\dot\alpha}}+1\,,$$

$$\mathsf{L}^{\alpha\beta}=\frac{1}{2}\Big(\lambda^\alpha\frac{\partial}{\partial\lambda_\beta}+\lambda^\beta\frac{\partial}{\partial\lambda_\alpha}\Big)\,,\qquad \mathsf{L}^{\dot\alpha\dot\beta}=\frac{1}{2}\Big(\tilde\lambda^{\dot\alpha}\frac{\partial}{\partial\tilde\lambda_{\dot\beta}}+\tilde\lambda^{\dot\beta}\frac{\partial}{\partial\tilde\lambda_{\dot\alpha}}\Big)\,;$$

$$\mathsf{Q}^{\alpha\,A}=\lambda^\alpha\eta^A\,,\qquad \tilde{\mathsf{Q}}^{\dot\alpha}_A=\tilde\lambda^{\dot\alpha}\partial_A,\qquad \mathsf{S}^\alpha_A=\lambda^\alpha\partial_A\,,\qquad \tilde{\mathsf{S}}^{\dot\alpha\,A}=\tilde\lambda^{\dot\alpha}\eta^A\,,\qquad \partial_A=\frac{\partial}{\partial\eta^A}\,,$$

$$\mathsf{R}^A{}_B=\eta^A\partial_B-\frac{1}{4}\delta^A_B\,\eta^C\partial_C\,,\qquad \Omega=-\frac{1}{2}\lambda^\alpha\frac{\partial}{\partial\lambda^\alpha}+\frac{1}{2}\tilde\lambda^{\dot\alpha}\frac{\partial}{\partial\tilde\lambda^{\dot\alpha}}+\frac{1}{2}\eta^A\partial_A\xrightarrow{\text{on-shell}}1\,.$$

- They satisfy the $\mathfrak{psu}(2,2|4)$ algebra. E.g.

$$\{\mathsf{Q}^{\alpha\,A},\mathsf{S}^\beta_B\}=\varepsilon^{\alpha\beta}\mathsf{R}^A{}_B+\delta^A_B\mathsf{L}^{\alpha\beta}-\frac{1}{2}\varepsilon^{\alpha\beta}\delta^A_B(\mathsf{D}-\Omega+1)$$

- Superconformal symmetry implies that the superamplitude is invariant under the action of these generators

$$\sum_j O_j\mathscr{A}_n(p_i,\eta_i)=0,\qquad O=\mathsf{P},\mathsf{K},\mathsf{D},\mathsf{L},\mathsf{Q},\mathsf{S},\mathsf{R}$$

Note $\Omega=1$ when acting on Ψ or $\mathscr{A}_n$.

Celestial superamplitude in $\mathcal{N} = 4$ SYM

- On-shell superfield $\rightsquigarrow$ Celestial superfield/superoperator :

$$\Psi_\Delta(z, \bar{z}, \eta) = \int_0^\infty d\omega\, \omega^{\Delta-1} \Psi(\varepsilon\omega q^\mu(z, \bar{z}), \eta) \ .$$

- In components, it reads ($h - \bar{h} = 1$)

$$\begin{aligned}\Psi_{h,\bar{h}}(z, \bar{z}, \eta) = \quad & \mathsf{G}^+(h, \bar{h}) + \eta^A\, \Gamma_A(h - \tfrac{1}{4}, \bar{h} + \tfrac{1}{4}) \\ & + \frac{1}{2!}\eta^A\eta^B\, \Phi_{AB}(h - \tfrac{1}{2}, \bar{h} + \tfrac{1}{2}) \\ & + \frac{1}{3!}\epsilon_{ABCD}\eta^A\eta^B\eta^C\, \bar{\Gamma}^D(h - \tfrac{3}{4}, \bar{h} + \tfrac{3}{4}) \\ & + \frac{1}{4!}\epsilon_{ABCD}\eta^A\eta^B\eta^C\eta^D\, \mathsf{G}^-(h - 1, \bar{h} + 1)\end{aligned}$$

- Celestial superamplitude

$$\mathscr{M}_n(h_i, \bar{h}_i, z_i, \bar{z}_i, \eta_i) = \langle \Psi_{h_1,\bar{h}_1}(z_1, \bar{z}_1, \eta_1) \cdots \Psi_{h_n,\bar{h}_n}(z_n, \bar{z}_n, \eta_n) \rangle \ .$$

Superconformal symmetry in celestial basis

- Introducing

$$\hat{\mathsf{k}}^{\alpha} = \begin{pmatrix} \partial_z \\ z\partial_z + 2h - 1 - \frac{1}{2}\eta^A\partial_A \end{pmatrix} e^{-\partial_h/2} , \qquad \hat{\tilde{\mathsf{k}}}^{\dot\alpha} = \begin{pmatrix} \partial_{\bar z} \\ \bar z\partial_{\bar z} + 2\bar h - 1 + \frac{1}{2}\eta^A\partial_A \end{pmatrix} e^{-\partial_{\bar h}/2} .$$

we then have sueprconformal generators in celestial basis

$$\widehat{P}^{\dot\alpha\alpha} = \tilde{\mathsf{p}}^{\dot\alpha}\,\mathsf{p}^{\alpha} , \qquad \widehat{K}^{\dot\alpha\alpha} = \hat{\tilde{\mathsf{k}}}^{\dot\alpha}\,\hat{\mathsf{k}}^{\alpha} , \qquad \widehat{D} = 1 - h - \bar h ,$$

$$\widehat{L}_{\alpha\beta} = \frac{1}{2}(\hat{\mathsf{k}}^{\alpha}\mathsf{p}^{\beta} + \hat{\mathsf{k}}^{\beta}\mathsf{p}^{\alpha}) , \qquad \widehat{L}_{\dot\alpha\dot\beta} = \frac{1}{2}(\hat{\tilde{\mathsf{k}}}^{\dot\alpha}\tilde{\mathsf{p}}^{\dot\beta} + \hat{\tilde{\mathsf{k}}}^{\dot\beta}\tilde{\mathsf{p}}^{\dot\alpha}) ;$$

$$\widehat{R}^{A}{}_{B} = \eta^A\partial_B - \frac{1}{4}\delta^A_B\eta^C\partial_C , \qquad \widehat{\Omega} = h - \bar h = J \xrightarrow{\text{on-shell}} 1 ;$$

$$\widehat{Q}^{\alpha\,A} = \mathsf{p}^{\alpha}\eta^A e^{-\frac{1}{4}(\partial_h - \partial_{\bar h})} , \qquad \widehat{\tilde{Q}}{}^{\dot\alpha}_{A} = \tilde{\mathsf{p}}^{\dot\alpha}\partial_A e^{\frac{1}{4}(\partial_h - \partial_{\bar h})} ,$$

$$\widehat{S}^{\alpha}_{A} = \hat{\mathsf{k}}^{\alpha}\partial_A e^{\frac{1}{4}(\partial_h - \partial_{\bar h})} , \qquad \widehat{\tilde{S}}{}^{\dot\alpha\,A} = \hat{\tilde{\mathsf{k}}}^{\dot\alpha}\eta^A e^{-\frac{1}{4}(\partial_h - \partial_{\bar h})} .$$

$$\sum_j \left\{ \varepsilon_j \widehat{P}_j,\ \widehat{L}^{\alpha\beta}_j,\ \widehat{L}^{\dot\alpha\dot\beta}_j,\ \varepsilon_j \widehat{K}^{\alpha\dot\beta}_j,\ \widehat{D}_j,\ (\widehat{Q}^{\alpha}_{A})_j,\ \varepsilon_j(\widehat{\tilde{Q}}{}^{\dot\alpha\,A})_j,\ (\widehat{R}^{A}_{B})_j,\ (\widehat{S}^{\alpha}_{A})_j,\ \varepsilon_j(\widehat{\tilde{S}}{}^{\dot\alpha\,A})_j \right\} \mathscr{M}_n = 0$$

- Celestial superamplitude = "superconformal correlator"

Standard SCFT: $Q^2 = P_{\text{2D}} = L_{\text{4D}}$ ↔ Celestial SCFT: $Q^2 = P_{\text{4D}}$

$$\mathscr{M}_n(h_i, \bar h_i, z_i, \bar z_i, \eta_i) = \langle \Psi_{h_1,\bar h_1}(z_1, \bar z_1, \eta_1) \cdots \Psi_{h_n,\bar h_n}(z_n, \bar z_n, \eta_n) \rangle_{\text{CCFT}}$$

$$= \left(\prod_{j=1}^{n} (cz_j + d)^{-2h_j} (\bar c\bar z_j + \bar d)^{-2\bar h_j} \right) \mathscr{M}_n\left(h_i, \bar h_i, \frac{az_i + b}{cz_i + d}, \frac{\bar a\bar z_i + \bar b}{\bar c\bar z_i + \bar d}, \left(\frac{cz_i + d}{\bar c\bar z_i + \bar d}\right)^{\frac{1}{2}} \eta_i \right) .$$

Asymptotic sym: Holographic chiral algebra from soft-soft OPE

Celestial operators: soft v.s. hard

- From now on,all particles are out-going $\epsilon = +1$.

$$\mathcal{O}_{\Delta,J}(z,\bar{z})$$

Here $\Delta \in 1 + i\mathbb{R}$ to form a basis, but can be analytically continue

- Soft operator: $\mathcal{O}_{k,J}$ with $k = |J|, |J| - 1, \cdots$ $\rightsquigarrow$ symmetry

  ★ Soft current:

$$R^{k,J}(z,\bar{z}) = \lim_{\Delta \to k} (\Delta - k)\mathcal{O}_{\Delta,J}(z,\bar{z})$$

- Hard operator: the rest

- Ambiguities arise when scattering **soft** particles of mixed helicities $\rightsquigarrow$ only consider **positive** helicity soft particles with $J > 0$

Ward identity (or conformally soft theorem)

- Soft theorem $\omega_s \to 0$:

$$\mathcal{A}_{n+1} \xrightarrow{p_s \to 0} \Big(\underset{\substack{\uparrow \\ \omega_s^{-n}}}{S^{(0)}} + \underset{\substack{\uparrow \\ \omega_s^{-n+1}}}{S^{(1)}} + \cdots \Big) \mathcal{A}_n$$

- Mellin transformation $\rightsquigarrow$ Ward identity [Fan, Fotopoulos, Taylor 19]

$$\int_0^\Lambda d\omega_s \; \omega_s^{\Delta_s - 1} \omega_s^{-n} = \frac{\Lambda^{\Delta_s - n}}{\Delta_s - n} \xrightarrow{\Delta_s \to n} \frac{1}{\Delta_s - n} + \cdots$$

Soft factor $\omega_s^{-n} \rightsquigarrow$ pole at $\Delta_s = n \rightsquigarrow$ current $H_n = \lim_{\Delta \to n} (\Delta - n) O_\Delta$

- E.g. the leading soft theorem for positive helicity gluon has soft factor

$$S^{(0)} = \sum_j \frac{p_j \cdot \varepsilon_s^+}{p_j \cdot p_s} T_j^a$$

which leads to Ward identity associated with KM current H^a

$$\lim_{\Delta \to 1} (\Delta - 1) \langle O^a_{\Delta,+1}(z, \bar{z}) \mathcal{O}^{b_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}^{b_m}_{\Delta_m, J_m}(z_m, \bar{z}_m) \rangle$$

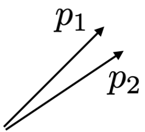
$$= \sum_j \frac{f^{a b_j c_j}}{z - z_j} \langle \mathcal{O}^{b_1}_{\Delta_1, J_1}(z_1, \bar{z}_1) \cdots \mathcal{O}^{c_j}_{\Delta_j, J_j}(z_j, \bar{z}_j) \cdots \mathcal{O}^{b_m}_{\Delta_m, J_m}(z_m, \bar{z}_m) \rangle$$

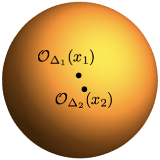
Celestial OPE

- In the collinear limit, $p_1 // p_2$, $P = p_1 + p_2$, and $P^2 \to 0$, the scattering amplitudes factorize

$$\mathcal{A}_n(1^{h_1}, 2^{h_2}, \cdots) \xrightarrow{p_1//p_2} \sum_h \text{Split}(1^{h_1}, 2^{h_2} \to P^h)\mathcal{A}_{n-1}(P^h, \cdots)$$

- Since $p_1 \cdot p_2 \propto (x_{12})^2$, the collinear limit of momenta just corresponds to the collision of two points on the celestial sphere $x_1 \to x_2$.





Celestial Sphere

- Performing the Mellin transformation on split function gives the celestial OPE.

Examples of celestial OPE in 4-dim

- E.g. the split function for gluons with positive helicity is

$$\text{Split}(1^+, 2^+ \to P^+) = \frac{1}{\sqrt{\gamma(1-\gamma)}} \frac{1}{\langle 12 \rangle} = \frac{\omega}{\omega_1 \omega_2} \frac{1}{\mathsf{z}_{12}}$$

where $p_1 = \gamma P$, $p_2 = (1-\gamma)P$. Mellin transformation gives

[Fan, Fotopoulos, Taylor 19]

$$\mathcal{O}^A_{\Delta_1,+}(\mathsf{z}_1, \bar{\mathsf{z}}_1)\mathcal{O}^B_{\Delta_2,+}(\mathsf{z}_2, \bar{\mathsf{z}}_2) \sim \frac{f^{ABC}}{\mathsf{z}_{12}} B(\Delta_1 - 1, \Delta_2 - 1)\mathcal{O}^C_{\Delta_1+\Delta_2-1,+}(\mathsf{z}_2, \bar{\mathsf{z}}_2)$$

- Similarly, the celestial graviton OPE is [Pate, Raclariu, Strominger, Yuan '19]

$$\mathcal{O}_{\Delta_1,+2}(z_1, \bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2, \bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2 - 1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2, \bar{z}_2)$$

- Supersymmetry: superfield $\to$ super-OPE [Jiang '21]
- Celestial OPEs can also be derived from string worldsheet. [Jiang '21]

Ingredient 1: Mode decomposition of soft current

- We work in 4-dim with **(2,2)** signature $\rightsquigarrow$ $z, \bar{z}$ are independent.
- Chiral enhancement of Lorentz symmetry:

$$SL(2,\mathbb{R}) \times \overline{SL(2,\mathbb{R})} \subset Vir \times \overline{SL(2,\mathbb{R})} \subset Vir \times \overline{Vir} = superrotation$$

- The soft currents admit decomposition into chiral currents

[Guevara, Himwich, Pate, Strominger '21]

$$R^{k,J}(z,\bar{z}) \;=\; \bar{z}^{J-k} R^{k,J}_{\frac{k-J}{2}}(z) + \bar{z}^{J-k-1} R^{k,J}_{\frac{k-J+2}{2}}(z) + \cdots + R^{k,J}_{\frac{J-k}{2}}(z)$$

Chiral currents $R^{k,J}_n(z)$ transform in the $(J+1-k)$-dim representation of $\overline{SL(2,\mathbb{R})}$.

- It is convenient to rescale the chiral currents: [Strominger '21]

$$\mathcal{R}^{i,J}_n = (i-1-n)!(i-1+n)! R^{J+2-2i,J}_n$$

$$n = 1-i,\, 2-i, \cdots i-1, \quad i = 1,\, \frac{3}{2},\, 2, \cdots$$

Light transformation of soft operator

$$\mathcal{R}_n^{i,J} = (i-1-n)!(i-1+n)! R_n^{J+2-2i,J}$$

Physically, this can be understood as light transformation: [Strominger '21]

$$\bar{\mathbf{L}}[\mathcal{O}]_{(h,1-\bar{h})}(z,\bar{w}) = \int d\bar{z}\ (\bar{w}-\bar{z})^{2\bar{h}-2}\mathcal{O}_{(h,\bar{h})}(z,\bar{z})$$

[Kravchuk, Simmons-Duffin '18]

Plugging mode expansion gives

$$\epsilon\bar{\mathbf{L}}[\mathcal{O}_{k+\epsilon,J}](z,\bar{w}) = \bar{\mathbf{L}}[R^{k,J}](z,\bar{w})$$

$$= \sum_{n=\frac{k-J}{2}}^{\frac{J-k}{2}} R_n^{k,J}(z) \int d\bar{z}\ (\bar{w}-\bar{z})^{k+\epsilon-J-2}\,\bar{z}^{-n+\frac{J-k-\epsilon}{2}}$$

As a result,

$$\boxed{\bar{\mathbf{L}}[\mathcal{O}_{J+2-2i,J}](z,\bar{z}) = \pi\mathrm{i}\frac{(-)^{2i}}{\Gamma(2i)}\sum_{n=1-i}^{i-1}\frac{\mathcal{R}_n^{i,J}(z)}{(-\bar{z})^{i+n}}}$$

Ingredient 2: Summing over $\overline{SL(2,\mathbb{R})}$ descendants

- General OPE takes the form:

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \frac{C_{\mathcal{O}_1\mathcal{O}_2}^{\mathcal{O}_P}\ \mathcal{O}_{\Delta_P,J_P}(z_2,\bar{z}_2)}{(z_{12}\bar{z}_{12})^{\frac{\Delta_1+\Delta_2-\Delta_P}{2}}(z_{12}/\bar{z}_{12})^{\frac{J_1+J_2-J_P}{2}}} + \cdots$$

where $\mathcal{O}_{\Delta_P,J_P}$ is primary, dots are descendants.

- Want to sum over *all* $\overline{SL(2,\mathbb{R})}$ descendants $\rightsquigarrow$ $\overline{SL(2,\mathbb{R})}$ OPE block

[Czech, Lamprou, McCandlish, Mosk, Sully '16]

$$\int_{\bar{z}_2}^{\bar{z}_1} d\bar{z}_3\ \mathcal{O}_{\bar{h}_P}(\bar{z}_3)\ \langle \mathcal{O}_{\bar{h}_1}(\bar{z}_1)\mathcal{O}_{\bar{h}_2}(\bar{z}_2)\tilde{\mathcal{O}}_{1-\bar{h}_P}(\bar{z}_3)\rangle$$

$$= \int_{\bar{z}_2}^{\bar{z}_1} \frac{d\bar{z}_3\ \mathcal{O}_{\bar{h}_P}(\bar{z}_3)}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2+\bar{h}_P-1}\,\bar{z}_{32}^{\bar{h}_2-\bar{h}_1-\bar{h}_P+1}\,\bar{z}_{13}^{\bar{h}_1-\bar{h}_2-\bar{h}_P+1}}$$

Summing over all $\overline{SL(2,\mathbb{R})}$ descendants

[Guevara, Himwich, Pate, Strominger '21; Jiang '21]

OPE with all $\overline{SL(2,\mathbb{R})}$ descendants

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \mathcal{N}^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2}\frac{\bar{z}_{12}^{N-M}}{z_{12}^{N+M}} \times$$
$$\int_0^1 dt\ \mathcal{O}_{\Delta_P,J_P}(z_2,\bar{z}_2+t\bar{z}_{12})\ t^{\Delta_1-J_1-M+N-1}(1-t)^{\Delta_2-J_2-M+N-1}$$

where

$$M = \frac{\Delta_1+\Delta_2-\Delta_P}{2}\ , \qquad N = \frac{J_1+J_2-J_P}{2}\ ,$$

$$\mathcal{N}^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2} = \frac{C^{\mathcal{O}_P}_{\mathcal{O}_1\mathcal{O}_2}}{B(\Delta_1-J_1-M+N,\ \Delta_2-J_2-M+N)}\ .$$

Holographic chiral algebra: soft-soft OPE

- Let's start with OPE between graviton and matter

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - J_1 + 1, \Delta_2 - 1)\mathcal{O}_{\Delta_1+\Delta_2,J_1}(z_2,\bar{z}_2)$$

- Applying the $\overline{SL(2,\mathbb{R})}$ descendant summation formula leads to

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} \int_0^1 dt\, \mathcal{O}_{\Delta_1+\Delta_2,J_1}(z_2,\bar{z}_2+t\bar{z}_{12})\, t^{\Delta_1-J_1}(1-t)^{\Delta_2-2}$$

- Taking both operators soft $\rightsquigarrow$ OPE between two soft currents

$$\Delta_1 \to k = J_1, J_1-1, \cdots, \qquad \Delta_2 \to l = 2,\ 1, \cdots$$

- Picking up specific chiral component in each soft current $\rightsquigarrow$ OPE between two chiral currents

$$\boxed{\mathcal{R}^{i,J_1}_n(z_1)\mathcal{R}^{j,+2}_m(z_2) \sim -\frac{2}{z_{12}}\Big(m(i-1)-n(j-1)\Big)\mathcal{R}^{i+j-2,J_1}_{n+m}(z_2)}$$

Holographic chiral algebra in supersymmetric EYM theory

The same procedure gives all the chiral OPEs in super-EYM theory:

$$
\begin{aligned}
\text{graviton} \;\rightarrow\quad \mathcal{H}^i_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m}(0)\;,\\
\text{gravitino} \;\rightarrow\quad \mathcal{I}^i_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{I}^{i+j-2}_{n+m}(0)\;,\\
\text{gluon} \;\rightarrow\quad \mathcal{K}^{i,a}_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{K}^{i+j-2,a}_{n+m}(0)\;,\\
\text{gluino} \;\rightarrow\quad \mathcal{L}^{i,a}_n(z)\mathcal{H}^j_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{L}^{i+j-2,a}_{n+m}(0)\;,\\
\mathcal{K}^{i,a}_n(z)\mathcal{K}^{j,b}_m(0) &\sim \frac{f^{abc}}{z}\mathcal{K}^{i+j-1,c}_{n+m}(0)\;,\\
\mathcal{K}^{i,a}_n(z)\mathcal{L}^{j,b}_m(0) &\sim \frac{f^{abc}}{z}\mathcal{L}^{i+j-1,c}_{n+m}(0)\;,\\
\mathcal{I}^{i}_n(z)\mathcal{K}^{j,a}_m(0) &\sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{L}^{i+j-2,a}_{n+m}(0)\;,\\
\mathcal{I}\mathcal{I},\mathcal{L}\mathcal{I},\mathcal{L}\mathcal{L} &\sim 0\;.
\end{aligned}
$$

[Guevara, Himwich, Pate, Strominger '21][Jiang '21]

Since this algebra is generated by infinite chiral currents, I will call it **holographic chiral algebra (HCA)**.

Holographic chiral algebra

$$\mathcal{H}^i_n(z)\mathcal{H}^j_m(0) \sim -\frac{2}{z}\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m}(0)$$

$$i,j{=}1,\tfrac{3}{2},2,\cdots, \quad n{=}1{-}i,2{-}i,\cdots,i{-}1, \quad m{=}1{-}j,2{-}j,\cdots,j{-}1$$

Using the formula

$$[A,B](z) = \oint_z \frac{dw}{2\pi i} A(w)B(z)$$

one gets the commutator

$$[\mathcal{H}^i_{n,k}\,,\ \mathcal{H}^j_{m,l}] = -2\Big(m(i-1)-n(j-1)\Big)\mathcal{H}^{i+j-2}_{n+m,k+l}$$

This is just the $w_{1+\infty}$ algebra, or more precisely the loop algebra of the wedge algebra of $w_{1+\infty}$ algebra. [Strominger '21]

- $\mathcal{H}^1_0$ is a central term: $[\mathcal{H}^1_0, \mathcal{H}^i_n] = 0$.
- The soft gravitons up to sub-sub-leading order, $\mathcal{H}^{\frac{3}{2}}_n$, $\mathcal{H}^2_n$, $\mathcal{H}^{\frac{5}{2}}_n$, generate the infinte tower of chiral currents $\mathcal{H}^i_n$ and thus also the HCA.

Infinite Ward identities from soft-hard OPE

Questions?

What is the implication of the ∞-dim symmetries?

Especially, what are the corresponding Ward identities?

Soft-hard OPE

- As before, we start with OPE between graviton and matter

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1-1,\Delta_2-J_2+1)\mathcal{O}_{\Delta_1+\Delta_2,J_2}(z_2,\bar{z}_2)\ ,$$

- then sum over all $\overline{SL(2,\mathbb{R})}$ descendants

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim -\frac{\bar{z}_{12}}{z_{12}} \int_0^1 dt\ \mathcal{O}_{\Delta_1+\Delta_2,J_2}(z_2,\bar{z}_2+t\bar{z}_{12})\ t^{\Delta_1-2}(1-t)^{\Delta_2-J_2}\ .$$

- Taking $\mathcal{O}_{\Delta_1,+2}$ soft: $\Delta_1 \to l = 2, 1\cdots$, while keeping $\mathcal{O}_{\Delta_2,J_2}$ hard

$\rightsquigarrow$ OPE between soft current and hard operator

$$\begin{aligned} &H^l(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \\ \sim\ &-\frac{\bar{z}_{12}}{z_{12}}\sum_{s=0}^{1-l}\frac{(\bar{z}_{12})^s}{s!}\bar{\partial}^s\mathcal{O}_{\Delta_2+l,+J_2}(z_2,\bar{z}_2)\ \frac{(-1)^{-s+1-l}}{(-s+1-l)!}\frac{\Gamma(\Delta_2-J_2+1)}{\Gamma(\Delta_2-J_2+l+s)} \end{aligned}$$

This gives the action of symmetry current on hard operators

Ward identities from soft-hard OPE

$$H^l(z_1,\bar z_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar z_2) \sim -\frac{\bar z_{12}}{z_{12}}\sum_{s=0}^{1-l}\frac{(\bar z_{12})^s}{s!}\partial^s\mathcal{O}_{\Delta_2+l,+J_2}(z_2,\bar z_2)\,\frac{(-1)^{-s+1-l}}{(-s+1-l)!}\frac{\Gamma(\Delta_2-J_2+1)}{\Gamma(\Delta_2-J_2+l+s)}$$

Applying this OPE in correlator leads to:

∞ Graviton Ward identities [Jiang '21]

$$\langle H^l(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle$$
$$= \sum_{k=1}^{m}\sum_{s=0}^{1-l}\frac{(\bar z-\bar z_k)^{s+1}}{z-z_k}\frac{(-1)^{-s-l}}{s!(-s+1-l)!}\frac{\Gamma(2\bar h_k+1)}{\Gamma(2\bar h_k+l+s)}$$
$$\times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_2,\bar z_2)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle$$

They are equivalent to infinite soft theorems in scattering amplitudes.

Ward identities up to sub-sub-leading order

[Adamo, Mason, Sharma '19; Puhm '19; Guevara '19...]

- leading order $l = 1$:

$$\langle H^1(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle = -\sum_{k=1}^{m}\frac{\bar{z}-\bar{z}_k}{z-z_k}\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+1,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

- sub-leading order $l = 0$:

$$\langle H^0(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle = \sum_{k=1}^{m}\frac{(\bar{z}-\bar{z}_k)^2}{z-z_k}\left[\frac{2\bar{h}_k}{\bar{z}-\bar{z}_k}-\bar{\partial}_k\right]\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

- sub-sub-leading order $l = -1$:

$$\langle H^{-1}(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle = -\frac{1}{2}\sum_{k=1}^{m}\frac{(\bar{z}-\bar{z}_k)^3}{z-z_k}\left[\frac{2\bar{h}_k(2\bar{h}_k-1)}{(\bar{z}-\bar{z}_k)^2}-\frac{4\bar{h}_k\bar{\partial}_k}{\bar{z}-\bar{z}_k}+\bar{\partial}_k^2\right] \times\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k-1,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

Shadow Ward identities

Shadow transformation in 2D CFT: [Osborn '12]

$$\widetilde{O}_{(1-h,1-\bar{h})}(w,\bar{w}) \;=\; \int d^2z\,(z-w)^{2h-2}(\bar{z}-\bar{w})^{2\bar{h}-2}O_{(h,\bar{h})}(z,\bar{z})$$

Shadow graviton Ward identities [Jiang '21]

$$\begin{aligned}
&\langle \widetilde{H}^l(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
=\;& \frac{(-1)^l\pi}{(3-l)!}\sum_{k=1}^{m}\sum_{s=0}^{1-l}\frac{(s+1)\Gamma(2\bar{h}_k+1)}{\Gamma(2\bar{h}_k+l+s)}\times(w-z_k)^l(\bar{w}-\bar{z}_k)^{l+s-2} \\
&\times\bar{\partial}_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+l,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}$$

Shadow graviton Ward identities

- leading order $l = 1$:

$$\langle \widetilde{H^1}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$
$$= \quad -\frac{\pi}{2}\sum_{k=1}^{m}\frac{w-z_k}{\bar{w}-\bar{z}_k}\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+1,+J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

- sub-leading order $l = 0$:

$$\langle\frac{3}{\pi}\widetilde{H^0}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$
$$= \quad \sum_{k=1}^{m}\Big[\frac{\bar{h}_k}{(\bar{w}-\bar{z}_k)^2}+\frac{\bar{\partial}_k}{\bar{w}-\bar{z}_k}\Big]\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

$\rightsquigarrow$ stress tensor Ward identity [Kapec, Mitra, Raclariu, Strominger '16]

- sub-sub-leading order $l = -1$:

$$\langle\widetilde{H^{-1}}(w,\bar{w})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$
$$= \quad \sum_{k=1}^{m}\frac{1}{w-z_k}\Big[\frac{2\bar{h}_k(2\bar{h}_k-1)}{(\bar{w}-\bar{z}_k)^3}+\frac{4\bar{h}_k\bar{\partial}_k}{(\bar{w}-\bar{z}_k)^2}+\frac{3\bar{\partial}_k^2}{\bar{w}-\bar{z}_k}\Big]$$
$$\times\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k-1,J_k}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle$$

Question?

How robust are the Ward identities?

EFT corrections?

Let us first derive some general results!

General spinning OPE

- In the collinear limit, $p_1 // p_2$, $P = p_1 + p_2$, amplitudes factorize

$$A_n(1^{s_1}, 2^{s_2}, \cdots) \xrightarrow{p_1//p_2} \sum_{s_3} \underbrace{A_3(1^{s_1}+2^{s_2}, -P^{s_3})\frac{1}{P^2}}_{\text{Split}(1^{s_1}+2^{s_2}\to P^{-s_3})} A_{n-1}(P^{-s_3}, \cdots)$$

- Three-point on-shell amplitude is fixed by locality and symmetry!

$$\begin{aligned}
\text{Split}(1^{s_1}+2^{s_2}\to P^{-s_3}) &= \frac{1}{P^2} A_3(1^{s_1}, 2^{s_2}, -P^{s_3}) \\
&\propto \frac{1}{\langle 12\rangle [12]} [12]^{s_1+s_2-s_3} [1P]^{s_1+s_3-s_2} [2P]^{s_2+s_3-s_1} \\
&\propto \frac{[12]^{s_1+s_2+s_3-1}}{\langle 12\rangle} (\sqrt{x})^{s_1+s_3-s_2} (\sqrt{1-x})^{s_2+s_3-s_1}
\end{aligned}$$

where $s_1 + s_2 + s_3 \geq 0$ and $p_1 = xP, p_2 = (1-x)P$.

General spinning OPE

- In terms of celestial variables,

$$\text{Split}(1^{s_1} + 2^{s_2} \to P^{-s_3}) \quad \propto \quad \frac{\bar{z}_{12}^{s_1+s_2+s_3-1}}{z_{12}} \omega_1^{s_2+s_3-1} \omega_2^{s_1+s_3-1} \omega_P^{-s_3}$$

- Mellin transformation gives the general celestial OPE arising from cubic interactions of spinning particles:

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim c_{J_1J_2J_3} \frac{\bar{z}_{12}^{J_1+J_2+J_3-1}}{z_{12}} \times B\Big(\Delta_1 + J_2 + J_3 - 1, \Delta_2 + J_1 + J_3 - 1\Big)\mathcal{O}_{\Delta_3,-J_3}(z_2,\bar{z}_2)$$

where $\Delta_3 = \Delta_1 + \Delta_2 + J_1 + J_2 + J_3 - 2$.

- Including all the $\overline{SL(2,\mathbb{R})}$ descendants: [Jiang '21][Himwich, Pate, Singh '21]

$$\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \frac{c_{J_1J_2J_3}}{z_{12}} \sum_{s=0}^{\infty} \frac{(\bar{z}_{12})^{J_1+J_2+J_3+s-1}}{s!} \times \bar{\partial}^s \mathcal{O}_{\Delta_3,-J_3}(z_2,\bar{z}_2)\; B(\Delta_1 + s + J_2 + J_3 - 1, \Delta_2 + J_1 + J_3 - 1)$$

General Ward identities in CCFT

General formula for Ward identities [Jiang '21]

$$\begin{aligned}
&\langle R^{l,J}(z,\bar z)\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle \\
= \; &\sum_{k=1}^{m}(-1)^{\nu(\nu_1+\cdots+\nu_{k-1})}c_{JJ_kJ'_k}\sum_{s=0}^{1-l-J_k-J'_k}\frac{(\bar z-\bar z_k)^{J+J_k+J'_k+s-1}}{z-z_k} \\
&\times\frac{(-1)^{(1-l-s-J_k-J'_k)}}{s!(1-l-s-J_k-J'_k)!}\frac{\Gamma(\Delta_k+J+J'_k-1)}{\Gamma(\Delta_k+J+J_k+2J'_k+l+s-2)} \\
&\times\bar\partial_k^s\langle\mathcal{O}_{\Delta_1,J_1}(z_1,\bar z_1)\cdots\mathcal{O}_{\Delta_k+l+J+J_k+J'_k-2,-J'_k}(z_k,\bar z_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar z_m)\rangle
\end{aligned}$$

where $\nu_i = 0, 1$ for bosonic and fermionc operators, respectively.

- Applicable to massless theories controlled by cubic interactions, including fermionic fields.

Now we can apply the general results to discuss the EFT corrections to graviton Ward identities...

EFT correction to soft graviton theorem

$$\mathcal{A}_{n+1} \xlongequal{p_s \to 0} \left(S^{(0)} + S^{(1)} + S^{(2)}\right)\mathcal{A}_n + \mathscr{O}(p_s^2)$$

- $S^{(0)}$ leading soft factor: exact
- $S^{(1)}$ sub-leading soft factor: no EFT correction, but has 1-loop quantum correction
- $S^{(2)}$ sub-sub-leading soft factor: EFT correction

[Elvang, Jones, Naculich '16]

$$\mathcal{A}_{n+1} \xlongequal{p_s \to 0} \left(S^{(0)} + S^{(1)} + S^{(2)}\right)\mathcal{A}_n + \tilde{S}^{(2)}\mathcal{A}_n + \mathscr{O}(p_s^2)$$

$$\tilde{S}^{(2)}\mathcal{A}_n = \sum_k g_k \frac{[sk]^3}{\langle sk\rangle}\tilde{\mathcal{A}}_n^{(k)}$$

EFT correction to graviton Ward identities

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,J_2}(z_2,\bar{z}_2) \sim \frac{\bar{z}_{12}^3}{z_{12}} B(\Delta_1+1, \Delta_2 - J_2 + 3)\mathcal{O}_{\Delta_1+\Delta_2+2,J_2-2}(z_2,\bar{z}_2)$$

The resulting Ward identity is

$$\begin{aligned}
&\langle H^l(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
= &\sum_{k=1}^{m}\sum_{s=0}^{-l-1} \frac{(\bar{z}-\bar{z}_k)^{s+3}}{z-z_k}\frac{(-1)^{-l-s-1}}{s!(-l-s-1)!}\frac{\Gamma(\Delta_k - J_k + 3)}{\Gamma(\Delta_k - J_k + l + s + 4)} \\
&\times \bar{\partial}_k^s \langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+l+2,J_k-2}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}$$

Non-trivial correction starts at sub-sub-leading order $l = -1$:

$$\begin{aligned}
&\langle H^{-1}(z,\bar{z})\mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle \\
= &\sum_{k=1}^{m} \frac{(\bar{z}-\bar{z}_k)^{3}}{z-z_k}\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{\Delta_k+1,J_k-2}(z_k,\bar{z}_k)\cdots\mathcal{O}_{\Delta_m,J_m}(z_m,\bar{z}_m)\rangle
\end{aligned}$$

$$\mathcal{A}_{n+1} \xlongequal{p_s\to 0} \left(S^{(0)} + S^{(1)} + S^{(2)}\right)\mathcal{A}_n + \sum_k g_k \frac{[sk]^3}{\langle sk\rangle}\tilde{\mathcal{A}}_n^{(k)} + \ldots$$

Conclusion

Summary and Outlook

- Global spacetime symmetry:
  Poincare, conformal, and superconformal in $\mathcal{N} = 4$ SYM
- Infinite dimensional asymptotic symmetry:
  Holography chiral algebra (soft-soft OPE)
- HCA $\rightsquigarrow$ infinite Ward identities (soft-hard OPE)
- A general formula for celestial OPE from the cubic interactions

symmetry+locality $\to$ celestial OPE $\to$ HCA $\to$ Ward Identity

- Negative helicity soft particles?
- Quantum corrections?
- Multiple soft theorems?
- An exact model for celestial holography? String theory?


i^+
$\mathcal{I}^+$
i^0
Thank you!
$\mathcal{I}^-$
i^-